Sidon

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Chapter 1

Basic Sidon sets results

Definition 1 (Sidon set)**.** A set $S \subseteq M$, where M is an additive commutative monoid, is called a Sidon set if for all $a, b, c, d \in S$, then $a + b = c + d \implies \{a, b\} = \{c, d\}$, that is, there are no repeated sums.

Theorem 2. *The empty set is a Sidon set.*

Theorem 3. *A singleton set is a Sidon set.*

Theorem 4. *If S is a Sidon set and $T \subseteq S$, then T is a Sidon set.*

Theorem 5. *If S is a Sidon set in a finite additive commutative group M, then $|S| \cdot (|S| - 1) \leq |M|$.*

Proof. Let $S = \{a_1, \dots, a_n\}$. Consider the differences $a_i - a_j$ for $i \neq j$. Since S is Sidon, theses differences are unique, and there are at most $|M|$ of them. By injectivity, the proof follows. □